Designing Knotted Textiles Based on Mathematical Diagrams

Nithikul Nimkulrat* and Janette Matthews**

* Estonian Academy of Arts, Estonia, nithikul.nimkulrat@artun.ee

** Loughborough University, UK, J.Matthews@lboro.ac.uk

Abstract

A textile practitioner-researcher and a textile practitioner-mathematician have collaborated to analyse textile knot practice and to explore the design possibilities of knotted textiles through mathematical diagrams. The collaboration has made use of century-old mathematical knot theory diagramming techniques to characterise knots and braids in the context of textile design today. This paper reveals the textile design process – the way in which diagrams were manipulated and utilized as a design tool to generate and visualise textile knot patterns.

Keywords: craft; knot; knot diagram; mathematics; textiles

Introduction

Knots have been used for thousands of centuries as ornamental elements in many cultures, and the "beginning of knot tying most likely preceded the evolution of mankind" (Jablan and Sazdanovic 2007, 375). In textiles, knotting or macramé is a craft technique commonly used for making three-dimensional artefacts. Loose ends are essential for knotting textile work, however from a topological perspective, "no string with free ends can be knotted" (Devlin 1999, 234). This contrast between mathematical and craft knots raises a question as to whether both types of knots share any similarities. Although knotting and representations of knots have widely been utilized in various art forms, little evidence has been found of investigations into knot theory and its links to aesthetics in textile structures or the materiality of textile artefacts. This is in contrast to other mathematical concepts, e.g. geometry.

This paper is based on a collaborative research project carried out by a textile practitioner-researcher (Nimkulrat) and a textile practitioner-researcher with a first degree in mathematics (Matthews). The project was initiated in 2013 when Nimkulrat came across mathematical knot diagrams that appeared as if they could be used as a way to visualise knotted structures. Although Nimkulrat studied mathematics at school level, topics did not include topology and in order to investigate possible links between mathematical knot diagrams and Nimkulrat's physical knotted textiles, collaboration was necessary. Research questions include: (1) whether craft and mathematical knots share comparable characteristics; (2) whether knot theory can examine the mathematical properties of knotted textile structures; and (3) how knot theory can facilitate the design and production of knotted textiles.

To answer the above questions is both to conduct collaborative research through design (Koskinen et al. 2011) and to understand the basics of mathematical knot theory. Since 2013, the findings of this research at various stages have been reported (e.g. Nimkulrat and Matthews 2013; 2014; 2016).

A Brief Overview of Mathematical Knot Theory

The study of knots in mathematics concerns the patterns of knots and the positions of strands but ignores physical properties, such as tightness, size and the shape of individual loops (Delvin 1998). A fundamental problem in knot theory is determining whether two knots are equivalent. A mathematical knot diagram is used to illuminate knot properties and determine equivalence, if it exists. A knot diagram is a projection of a knot which appears as a simple line drawing that indicates the knot pattern. Lines are broken to show where the knot crosses itself and where a strand passes under or over. Figure 1 shows two mathematically equivalent knots. Both diagrams are representations of the same knot as the loop in Figure 1b may be removed to form the original figure-eight knot in Figure 1a.

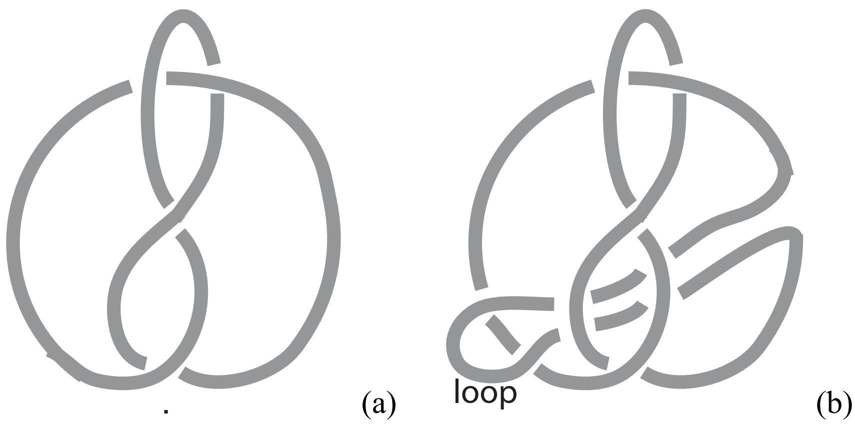


Figure 1. Two mathematically equivalent knots; both are figure-eight knots. In mathematical knot theory, the shape of individual loops is ignored when determining the equivalence of two or more knots. Diagrams: Janette Matthews.

The Design Collaboration between a Mathematician and a Textile Practitioner

The project is research through design that utilises a collaborative approach of textile knotting and mathematical knot theory. In this project, the role of Nimkulrat is a textile practitioner-researcher and that of Matthews is a mathematician. The collaboration started with an examination of a single craft knot used in Nimkulrat's installation *The White Forest* made from paper string (Figures 2a and 2b).

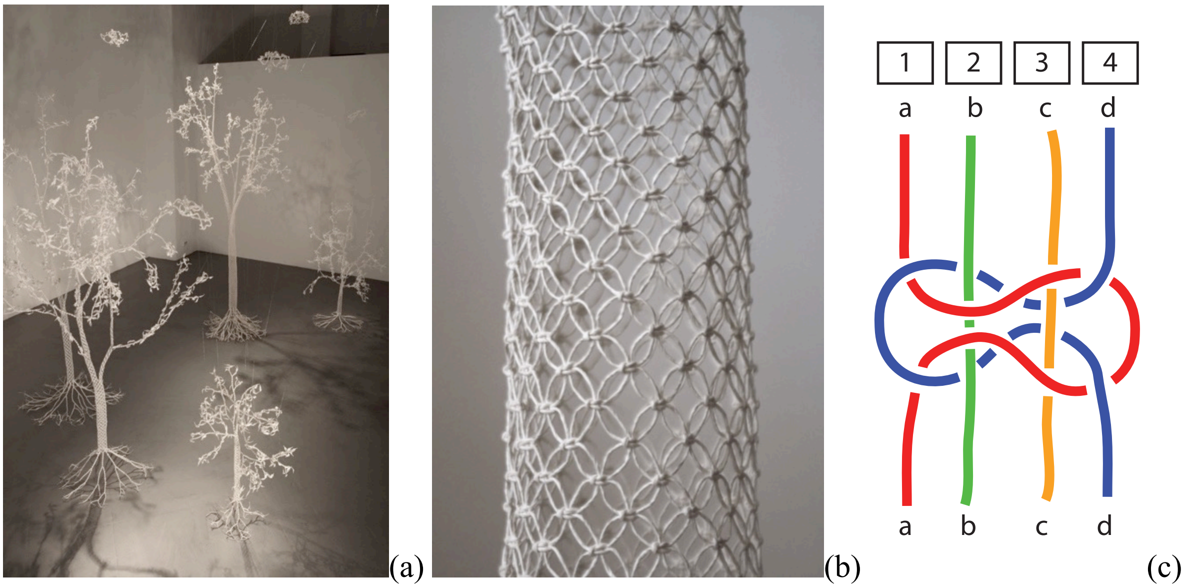
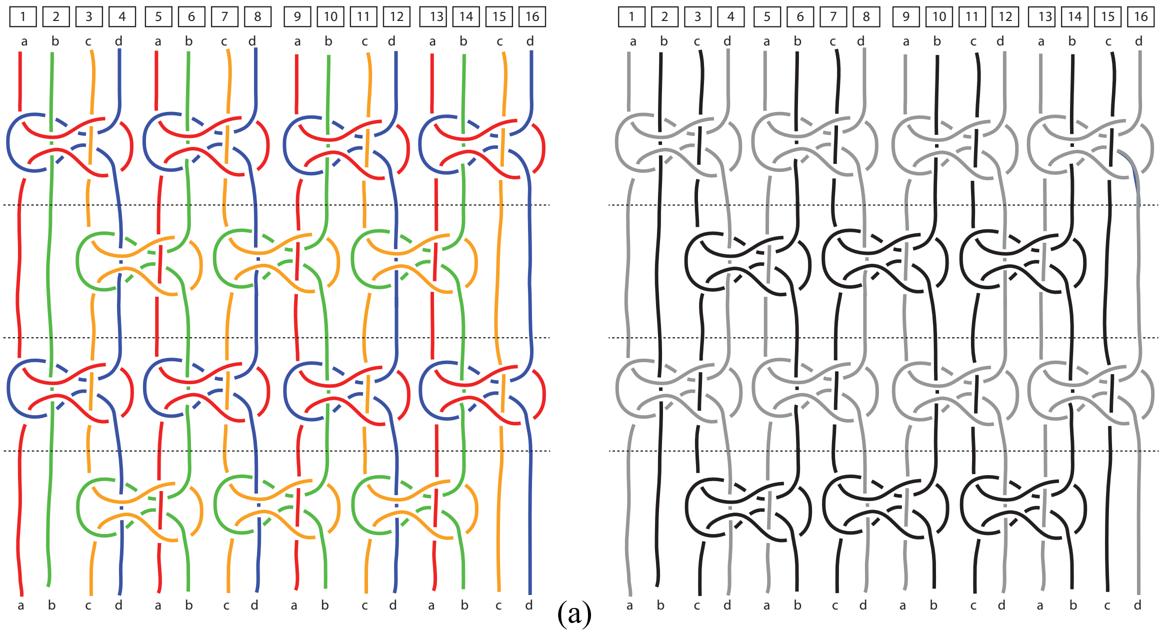


Figure 2. *The White Forest* installation (a); detail of the tubular knotted structure (b); and the knot diagram of a single knot (c). Photographs: Nithikul Nimkulrat. Diagram: Janette Matthews.

Matthews employed the knot diagram and its colourable property to characterise Nimkulrat's single knot. The diagram (Figure 2c) revealed that the positions of strands finish in the same place after tying a knot as where they started. Strand *a* starts and ends in position 1. Likewise, strand *b* remains in position 2, *c* in 3 and *d* in 4. This property was not obvious in Nimkulrat's craft practice (Figure 2b). From the positions of strands in this diagram, it can be implied that by changing positions of strands there is the potential to create various knot patterns.

The next stage was to examine whether a group of knots may be analysed using mathematical diagrams following the same principles for the characterisation of a single knot, and how this in turn might facilitate pattern design and the production of knotted textiles. Matthews utilised the same method to characterise a group of knots starting with four knots, each requiring four strands. In colouring the diagram, four colours were used alternately as in the characterisation of the single knot (Figure 3a). The first row of knots uses strands *a* (red) and *d* (blue) for knotting (active strands), while strands *b* (green) and *c* (yellow) are not touched (passive strands). The knots on the second row have *b* (green) and *c* (yellow) as active strands. The active strands from the previous row, *a* (red) and *d* (blue), are now passive. The third row uses the same strands as the knots on the first row and the fourth the same as the second. Once again, all strands finish in the same positions as they started.



(a) (b)

Figure 3. The diagram of multiple knots, using four colours (a) and two colours (b). Diagrams: Janette Matthews.

As textile practitioners, both Nimkulrat and Matthews instantly recognised pattern repeats in the diagram, e.g. every alternate row utilising the same strands for tying. This implied two possibilities for designing repeating knot patterns: (1) through the use of colours and (2) through changing active and passive strands.

To explore the first option, Matthews reduced the number of colours to two, using black and grey to re-colour the diagram – black replacing green (*b*) and yellow (*c*) wheras grey replacing red (*a*) and blue (*d*) (Figure 3b). On examining Figure 3b, it became obvious that the colour of the knot is the colour of the active strands, e.g. a black knot is created when the strands tying it are black. A further visible aspect is that the passive strands that link the knots would form either black or grey rounded rectangles – not a mixture of the two colours.

To verify this design possibility, Nimkulrat followed the diagram, using black and white paper string to make physical knots (white string was used instead of grey in Figure

3b). The actual knotted sample (Figure 4a) showed that across a row of knots (1) all active strands are one colour, black or white, and all passive strands are the other colour and (2) the colour of each knot in that row is the same colour as that of the active strands. It also revealed that white and black strands are active alternately in the tying of knots, and eventually black and white circles became apparent. The material properties of paper string, its stiffness in particular, transformed rounded rectangles in the diagram into circles in the sample.

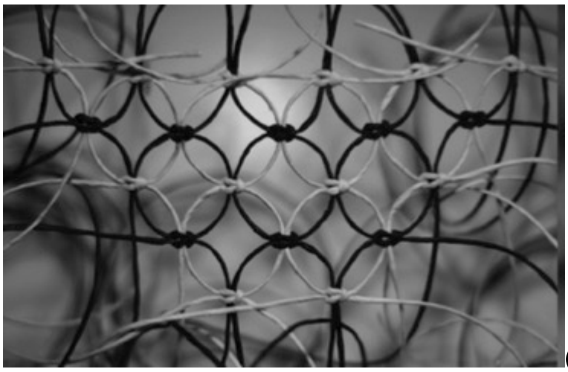
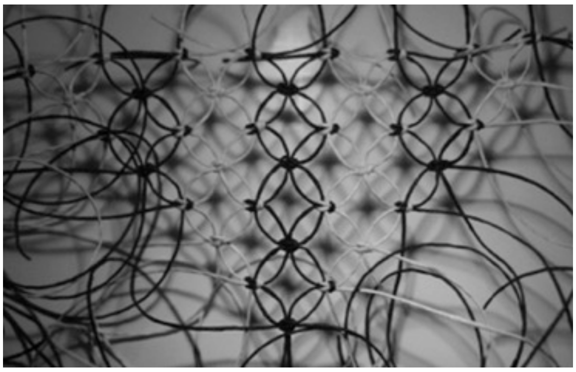

(a)


Figure 4. The physical knots based on the diagram on Figure 3b appear as a circle pattern (a) and the repositioning of strands results in a striped knot pattern (b). Photographs: Nithikul Nimkulrat.

In the next material experimentation, Nimkulrat explored the second possibility for designing repeating knot patterns – through changing active and passive strands. She repositioned strands in the following order: (from left to right) black-black-white-white and white-white-black-black. In this case both black and white strings were simultaneously active in tying the knot along the first row, and in the next row four strings of the same colour formed a solid colour knot, black or white. The process continued alternately between a row of mixed colour knots and a row of pure black or white ones. This iterative process led to a striped knot pattern (Figure 4b).

Subsequently, Matthews examined the sample in Figure 4b through the diagrammatic method to determine whether the method could be used to predict the knot pattern. The diagram revealed the alternating vertical columns, mixed-coloured knots – solid-grey knots, mixed-coloured knots and solid-black knots – and the alternating horizontal rows – mixed-colour knots and a row of solid alternating colour knots (Figure 5).

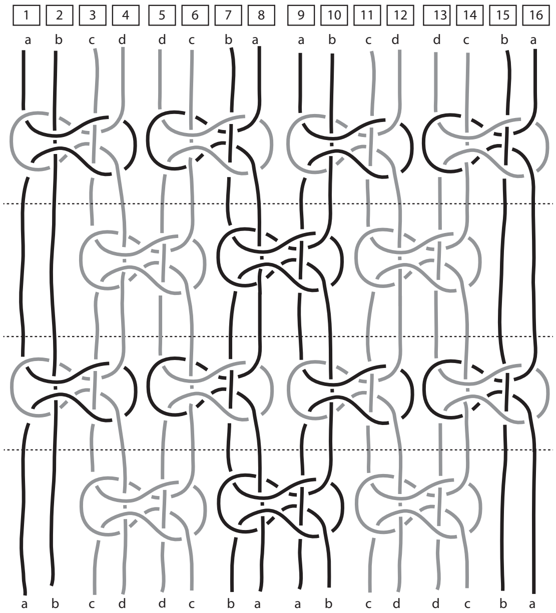


Figure 5. The knot diagram represents the physical striped knot pattern in Figure 4b. Diagram: Janette Matthews.

Reflection and Conclusion

Within this research collaboration, pattern development for knotted structures can be explored, predicted and modelled through the use of mathematical knot diagrams. New understanding of pattern in relation to the active/passive nature and positions of colour strands inspired new experimentation. Re-colouring the diagram showed a variety of design possibilities that were achievable by altering the positions of the strands' colours. This approach provides a way of visualising two-tone knot patterns prior to knotting.

Previously Nimkulrat had desired to use more than one colour in her textile knot practice, but had not made any attempts to do so. Although her knotting skills are advanced, experimenting with colours in the making of knotted textiles without the diagrammatic approach seemed too difficult to handle.

Based on the striped knot pattern (Figures 4b and 5), Nimkulrat used paper string to knot a textile artefact in a functional form of an armchair entitled *Black & White Striped Armchair* (Figure 6).


(a)

(b)

Figure 6. *Black & White Striped Armchair* a knotted textile artwork in the striped pattern. Photographs: Nithikul Nimkulrat.

The use of diagrams to show the process of making textiles is not new. What is novel here is the application of mathematical knot diagrams to describe craft knots, i.e. simple, broken lines to depict the positions of strands and the resulting knot pattern. The diagrams used in this paper are not graphical illustrations of textile knots, but mathematical representations. The use of colour in diagrams makes the positions and roles of strands in a knotted structure explicit and leads to an exploration of knotted pattern designs that may not have occurred otherwise. The paper has illuminated the role of mathematics in characterising and understanding craft knot practice. It has demonstrated the significance of cross-disciplinary collaboration on the development of textile practice.

The collaboration takes time and effort to share insights from each of the collaborators' perspectives so that, for example, Matthews could produce textile knots and Nimkulrat understood the application of relevant mathematics. A common language to communicate research findings and novel ideas is developed, including for example the use of maquettes, coloured knot diagrams, photographs and artefacts. This is an example of a research through design project in which cross-disciplinary collaboration is key.

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